SARVAJANIK UNIVERSITY
PH. D. Entrance Examination Syllabus/2021-22
MATHEMATICS

Unit-1. Advanced Linear Algebra: Linear operator Functions, linear operators, null space and range, rank and nullity theorem, operator inverses, application to matrix theory, computation of null space and range of a matrix, matrix of an operator, change of basis and similar matrices, Inner Product Spaces, Definitions and examples, Orthogonal sets, Fourier coefficients and partial identity, gram-schmidt process, QR factorization, Approximation and Orthogonal projection, equivalence of the problems, Computations using orthogonal and non-orthogonal sets, normal equations, projection operators, $M^{\perp}$. Orthogonal complements, Decomposition of the vector space, applications to an approximations and matrix theory, The Gram Matrix and Orthogonal Change of Basis, matrix representation of an inner products, orthogonal change of bases, rank of gram matrix.

Unit 2. Advanced Abstract Algebra: Group Theory; Conjugate of an element, class equation, Cauchy theorem, First part of Sylow's theorem, Third part of Sylow's theorem, Application of Sylow's theorem, Direct product of a group. Ring Theory; A particular Euclidean ring, Fermat's theorem, polynomial rings, primitive polynomials, Guass lemma, the Eisenstein criterion, polynomial rings over commutative rings, unique factorization domain, Field Theory; Extension fields, Finite extension field, Algebraic extension, Algebraic number, Roots of polynomials, splitting fields, Uniqueness of Splitting fields, construction with Straightedge and compass, More about roots, Simple extension, Fixed fields, Elementry symmetric functions, normal extension, Galois group, The fundamental theorem of Galois theory.

Unit-3. Complex analysis: Algebra of complex numbers, the complex plane, polynomials, power series, transcendental functions such as exponential, trigonometric and hyperbolic functions, Analytic functions, Cauchy-Riemann equations, Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem, Taylor series, Laurent series, calculus of residues, Conformal mappings, Mobius transformations.

Unit-4. Functional Analysis: Metric Space, Normed Space, Banach Space, Further Properties of Normed Spaces, Finite Dimensional Normed Space and Subspaces, Compactness and Finite Dimension Linear Operators; Bounded and Continuous Linear Operators, Linear Functionals,

Linear Operators and Functionals on Finite Dimensional Spaces, Normed Spaces of Operators, Dual space.

Unit-5. Topology: Basic concepts of topology, bases, sub-bases, subspace topology, order topology, product topology, quotient topology, metric topology, connectedness, Sequentially Compact Metric Space, Bolzano-Weierstrass Property (BWP),Totally bounded Space, Ascoli's theorem, countability and separation axioms.

Unit-6. Numerical Analysis: Transcendental Equation, Bisection method, Iteration method based on first and second degree equation, Rate of convergence, General iteration Methods, System of nonlinear equations, Methods for complex roots. Polynomial equations, Choice of iterative method. Direct Methods, Error analysis for direct root methods, Iteration methods. Eigen values and Eigenvectors, Bound on Eigen values, Jacobi method for symmetric matrices, Givens method for arbitrary Matrices, Power method, Inverse power method. Lagrangian and Newton interpolations, Finite difference operators, Interpolating polynomials, Hermite interpolation, Piecewise and Spline interpolations.

Unit-7. Advanced Calculus: Functions of two or more variables, continuity, directional derivatives, partial derivatives, total derivative, maxima and minima, saddle point, method of Lagrange's multipliers; Double and Triple integrals and their applications to area, volume and surface area, Vector Calculus; gradient, divergence and curl, Line integrals and Surface integrals, Green's theorem, Stokes' theorem, and Gauss divergence theorem.

Unit-8. Transform Calculus: Introduction to Laplace transform; Definition and properties, Inverse Laplace Transform, Laplace Transform of derivatives and integrals, Laplace Transform of some special functions, Application of Laplace Transform to Ordinary Differential Equations and Integral Equations, Fourier Series, Introduction to Fourier Transforms; Definition and properties, Fourier Sine and Cosine transforms of different functions, Parseval's Identity for Fourier Sine and Cosine Transforms, Application of Fourier Transform to Ordinary Differential Equations and Integral Equations, Introduction to Z-transform; Definition and properties.

Unit-9. Ordinary Differential Equations: Basic concepts and Linear Equations of the first order, Formation of Differential Equations (DE), Classification of DE, Initial and Boundary Value Problems, Definition of Solutions of DE, First Order Linear Equation, Exact Equations, Separable Equations. Linear Differential Equations of Higher Order; Higher Order Differential Equations,

Linear Independence, Equations with Constant Coefficients, Equations with Variable Coefficients Wronskian, Variation of Parameters, Some Standard Methods, Solutions in Power Series; Second Order Linear Equations with Ordinary Points, Legendre Equation and Legendre Polynomials, Second Order Equation with Regular Singular Point, Properties of Bessel Functions.

Unit-10. Partial Differential Equations: Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs. Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.